LISA Orbits

Formation stability

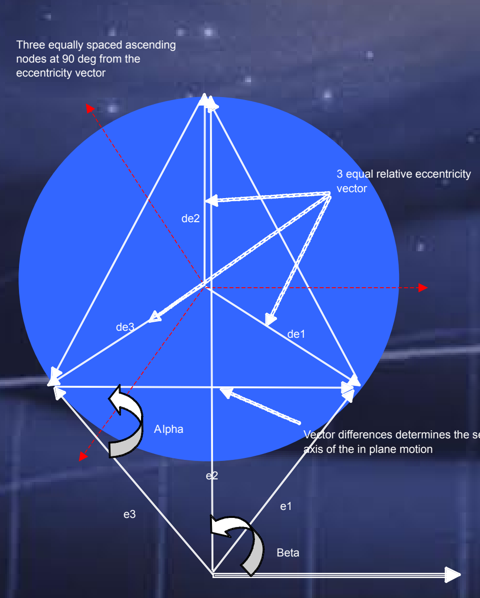
The LISA baseline formation is composed by 3 spacecraft in equilateral triangle formation; each arm measures 5million km, and the formation centre is 20 degrees away from Earth. The nominal mission time is 3.5 years, likely to be extended to 8.5 years.
Science operations ask for some stability in the formation, which has lead to formulate the following requirements:

- Relative range rate less than 15 m/s
- Range deviation between two arm lengths divided by sum of lengths less than 1% (typically 100000km)
- Breathing angle between the arms of the formation less than 3 deg (from side to side, i.e. roughly 1.5 degrees on each direction)

In the absence of any external perturbations linear theory predicts circular motion (thus constant inter-satellite range and zero range rate); when second order terms are considered, the eccentricity term results in a small distortion from pure circular motion.
The real factor affecting the stability is the presence of a bias term (in the orbital frame) from the effect of Earth's gravity: this term gives rise to significant secular effects, superimposed to which will be one year periodic terms.

Relative motion could be controlled by station keeping, but this solution is not preferred as would significantly interrupt science operations (even small manoeuvres performed by low thrust take several days). Another option is to optimise the formation design in order to reduce the effect of gravity induced drift: this can be achieved by:

- Locating the eccentricity vectors in an optimal location with respect to Earth's eccentricity vector
- Applying biases in initial velocity and position to minimise drift over the mission period



The formations considered have 3 equal differential eccentricity vectors, and 3 equally spaced ascending nodes. Only two angles are necessary to describe any of these formations, as Figure 1 shows. Anyhow, the fact that still 20 variables have to be optimised (9 initial velocity offsets, 9 initial position offsets and the 2 angles) and the highly multimodal character of the problem have made use of a genetic algorithm the preferred choice.

Several formation radii and Earth offsets have been investigated; also, the breathing angle and the range rate have been alternatively optimised. Table 1 summarises the results for the breathing angle optimisation.

Formation radius (km)	Earth offset (deg)		
	21 deg	23 deg	25 deg
2.00E+06	1.36	0.96	0.8
3.00E+06	1.33	1.05	0.93
5.00E+06	1.64	1.4	1.25

Table 1: breathing angle for different formations

The limit of 3deg is easily satisfied but some of the solutions are not compliant with the constraint on the range rate as Table 2 shows. Table 2 also presents the ranges for baseline (5 million km) formations at different Earth offsets.

Earth offset (deg)	max Earth range (km)	max excursion (deg)	max range rate (m/s)
21	63499482	1.64	15.77
23	68678518	1.40	15.99
25	73854182	1.25	13.91

Table 2: Earth range and maximum range rate for baseline formations at different Earth offsets

The range rate has proved to deliver the most requiring constraint, so it has been adopted as optimisation goal in the process of looking for the optimal formation. The resulting ephemeris for the optimal leading and trailing formations are given in Table 3 and Table 4. As it can be noticed the ephemeris sets are quite similar, which is natural as the problem is expected to be symmetrical.

	a(m)	e	incl(deg)	w(deg)	ascen(deg)	manom_0 (deg)
LISA1	1.49398E+11	0.008356	0.957483	60	144.6861	-224.6861
LISA2	1.49398E+11	0.014473	0.957483	90	24.6861	-134.6861
LISA3	1.49398E+11	0.008356	0.957483	-240	264.6861	-44.6861

Table 3: Ephemeris for optimal trailing target formation. r_formation=5e6km, Earth offset=20deg

	a(m)	e	incl(deg)	w(deg)	ascen(deg)	manom_0 (deg)
LISA1	1.49798E+11	0.008348	0.956218	61.7207	128.9015	-165.6222
LISA2	1.49798E+11	0.014203	0.956218	90.0000	8.9015	-73.9015
LISA3	1.49798E+11	0.008348	0.956218	-241.7207	248.9015	17.8192

Table 4: Ephemeris for optimal leading target formation. r_formation=5e6km, Earth offset=20deg

The resulting graphs for the range and range rate variations can be seen in Figure 2, Figure 3, Figure 4 and Figure 5.

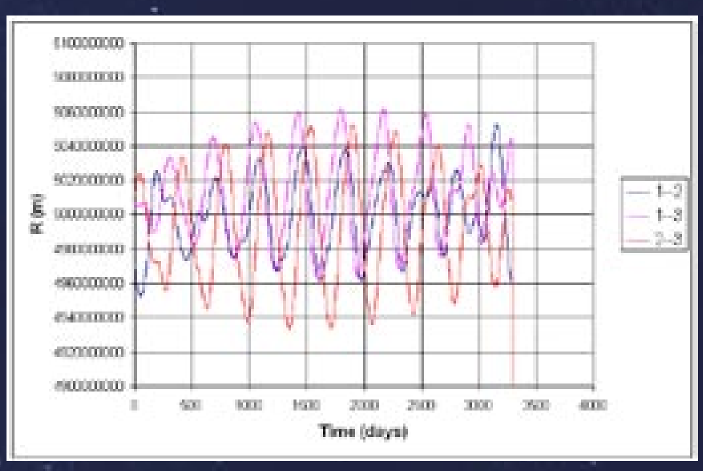


Figure 2: range variation for optimal trailing formation. r_formation=5e6km, Earth offset=21deg

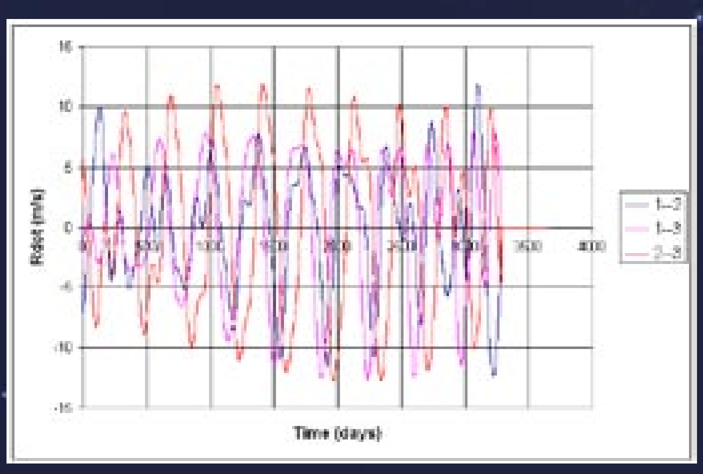


Figure 3: range rate variation for optimal trailing formation. r_formation=5e6km, Earth offset=21deg

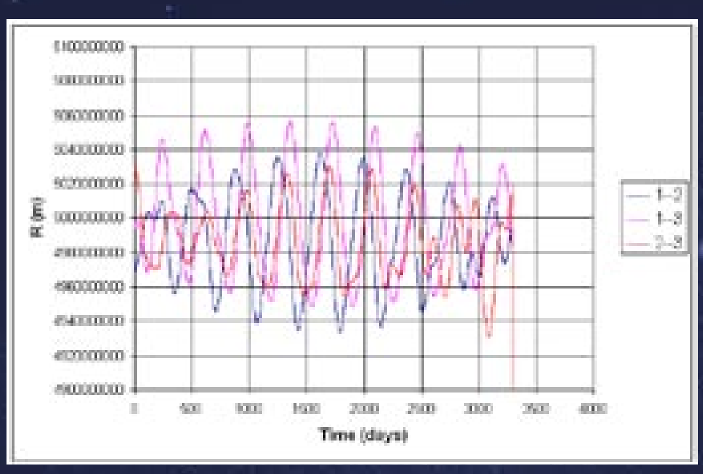


Figure 4: range variation for optimal leading formation. r_formation=5e6km, Earth offset=21deg

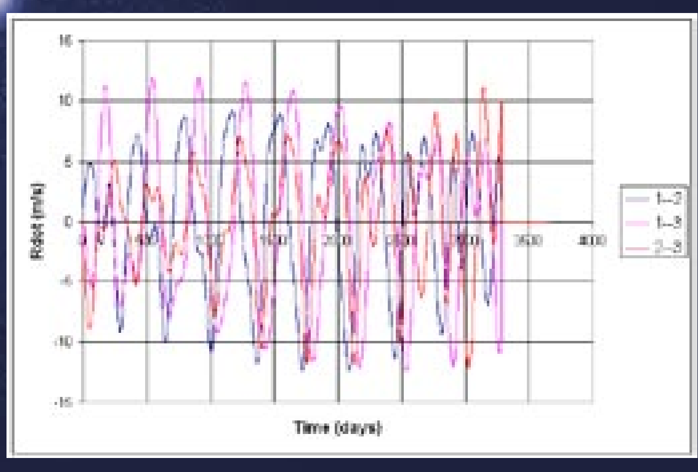


Figure 5: range rate variation for optimal leading formation. r_formation=5e6km, Earth offset=21deg
In both cases the range rate can be limited efficiently to a peak of about 13m/s, while the breathing angle doesn't exceed 1.8 degrees.

Self gravity acceleration

When the mass distribution of each one of the 3 science-craft is not perfectly symmetric (which is not possible in real life) a perturbing acceleration arises on each science-craft. It should be possible to limit this effect to an acceleration of 1e-9 m/s2, on each science-craft, which means that in the worst-case scenario the relative acceleration between two science-craft will be 2e-9 m/s2. It is of absolute interest to figure out whether this perturbation can significantly affect the stability performance of the formation, and for this reason some analysis has been assessed. In modelling the problem with perturbing accelerations it has been assumed that only one science-craft is affected by the disturbance, and that the disturbance is acting on the imaginary line connecting the accelerated science-craft to one of the other two. This is not what happens in reality, but it is comparable to that, as what is important is the relative (to the two other science-craft) motion, not the absolute motion. For the results obtained in this model to be comparable to what happens in reality it is necessary to apply a stronger perturbation than the one actually produced on a single science-craft.

Table 5 summarises what happens if the perturbation is applied to the optimal formation. In the specific case the perturbation is acting on spacecraft number 2, in the direction towards spacecraft number 1.

perturbing acceleration (m/s/s)	angular excursion (deg)	max rdot12 (m/s)	max rdot13 (m/s)	max rdot23 (m/s)
0	1.64	14.95	15.77	11.69
2.00E-09	1.65	14.26	15.77	11.69
6.00E-09	1.68	12.99	15.77	11.93
1.00E-08	1.71	12.72	15.77	12.92
3.00E-08	2.33	14.97	15.77	19.73
5.00E-08	3.36	23.29	15.77	31.96
1.00E-07	6.32	56.89	15.77	67.82

Table 5: results for different levels of perturbation acting on the optimal baseline formation; perturbation acting on sc2 in the direction towards sc1

Results show how the formation is quite robust to this perturbation, surely for the expected value of self-gravity acceleration. The results also show a non linearity of the problem: the obtained range rates don't match what would be expected from a rectilinear motion with constant acceleration (for instance an acceleration of 1e-8m/s2 would produce about 3m/s change in the range rate over 9 years); that happens because the acceleration applied is fixed, but in a reference frame that is rotating (motion of the formation around its centre, 1 revolution in 1 year) around another rotating reference frame (heliocentric motion of the centre of the formation, again 1 revolution in 1 year).

Anyhow the stability can be further improved if the information on the perturbation direction is included in the optimisation process, as Table 6 shows.

perturbing acceleration (m/s/s)	angular excursion (deg)	max rdot12 (m/s)	max rdot13 (m/s)	max rdot23 (m/s)
0	1.64	14.95	15.77	11.69
1.00E-08	1.52	15.41	20.75	14.64
3.00E-08	1.67	12.14	20.68	11.56
5.00E-08	2.13	22.24	26.81	26.05
1.00E-07	3.32	40.07	26.38	38.91

Table 6: results for optimisation assuming different levels of perturbation; r formation=5e6 km, Earth offset=21deg, perturbation acting on sc2 in the direction towards sc1

The consistent range rates are not too scary, as in these specific optimisations the breathing angle has been minimised, regardless of the range rates.

Transfers to the formation

The 3 LISA spacecraft are launched together, nominally on a Delta IV launcher. The launcher is able to inject the stack into a low escape velocity orbit.
The transfer to the required offset from Earth starts with a change in the semi-major axis: the change is mostly supplied by the launcher, but it can be supplemented by a Δv manoeuvre for each individual spacecraft. The drift is stopped when the target offset is reached, by restoring the semi-major axis to 1AU. An out of the ecliptic manoeuvre is also necessary, as the individual target orbits all have an inclination of approximately 1 degree in the baseline formation.

Figure 6 gives an example on how the 3 heliocentric transfers look like.

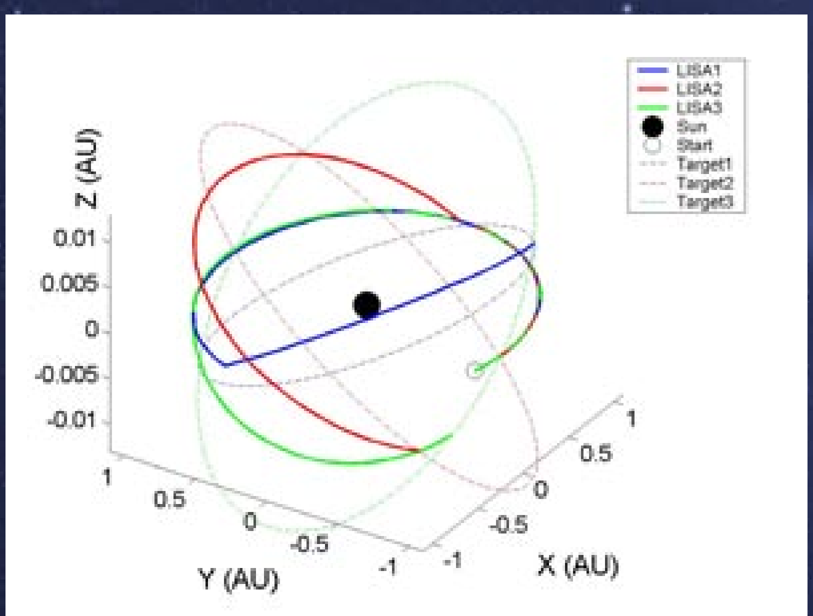


Figure 6: transfer to LISA formation example

Transfer time is constrained to be less than 14 months. The total dv for the 3 spacecraft should not exceed 3km/s, with every spacecraft below 1.1km/s.

The nominal propulsion system is chemical.

The number of controls to be optimised (more than 20 parameters) and the type of problem suggested the use of a genetic algorithm.

Both for trailing and leading baseline formations it turns out to be possible to meet the dv requirements, as optimal transfers require a total dv of about 2.7km/s, with a January (Earth at pericentre) launch for a leading formation, July launch for a trailing formation.

If a year-round launch capability is looked for, it is not possible to meet the constraints for the whole year, as Table 7 and Table 8 show.

Launch	v inf	dv1	dv2	dv3	total dv
24-Aug-14	707	816	999	853	2668
01-Jan-15	544	1206	989	966	3161

Table 7: Best case/ worst case for trailing target formation

Launch	v inf	dv1	dv2	dv3	total dv
22-Jul-14	560	1040	1200	1278	3518
20-Jan-15	653	907	1020	838	2765

Table 8: Best case/ worst case for leading target formation

An interesting thing to be noticed is that the transfer problem is not symmetrical: there is a small difference on the total dv for the best solutions, and the difference increases even further when considering the worst cases. This happens because transfers to leading formations fly on faster orbits than transfers to trailing formations (which implies added cost on plane change and pericentre rotation manoeuvres), but can also be forecasted by simple 2D models.

A possible solution to this problem lies in targeting smaller radii formations, as the formation radius determines the inclination of the target orbits. Targeting a 2million km, for instance, would provide a 26% saving on the best-case dv and a 20% on the worst case, as Table 9 shows

Launch	v inf	dv1	dv2	dv3	total dv
23-Aug-14	729	621	701	638	1960
01-Jan-15	492	926	786	797	2509

Table 9: Best case/ worst case for trailing target formation; formation radius=2e6km

If it is desired to stick to the baseline 5 million km formation, one possibility is to follow a strategy where during one part of the year the trailing formation is targeted while during the other part the leading formation is looked for. Table 10 summarises the results, highlighting how this strategy delivers a worst-case dv of 2928m/s for a late November launch; the worst-case single dv is 1109m/s, which is just above the desired boundary of 1100m/s.

Launch	v inf	dv1	dv2	dv3	formation type	total dv
23-Feb-14	707	983	984	879	leading	2846
10-Apr-14	633	896	899	1109	trailing	2904
28-May-14	659	885	918	938	trailing	2741
01-Jul-14	649	887	1062	832	trailing	2781
24-Aug-14	707	816	999	853	trailing	2668
10-Oct-14	713	1001	984	863	trailing	2848
25-Nov-14	661	931	1013	984	leading	2928
20-Jan-15	653	907	1020	838	leading	2765

Table 10: summary of year-round launch capability to target baseline formation

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